

Research on volatility and extreme risk in Canadian oil companies' stock price

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Abstract: The Canadian oil market is one of the most essential industries in Canada and has played a crucial role in international finance and the economy. The volatility in stock price reflects the risks a stock faces and therefore, studying Canadian oil stock price volatility is a must. Besides, the data period also consists of some extreme periods such as subprime mortgages in 2007 and the pandemic of COVID-19 in 2020. As a result, the extreme risks will be studied in this research. This research carried out some models and the ARMA-TGARCH model fits the data best. When calculating the predicted values, ARMA-TGARCH could not give accurate estimations, especially in long-term forecasting. Then the EVT-POT is used and outputs a relatively high value at risk. This indicates some implications and advice for investors and stock traders. The whole research proves that basic time series models and extreme value theory could not capture real-life problems. Risk predictions should be studied case by case and together with behavior economics such as preference, history, utility, etc. This study also claims that risk analysis should not depend on lag terms only. Some real-life factors should be added to this topic.

Keywords: oil market, volatility, extreme risks, ARMA-TGARCH,

EVT-POT, investment

JEL Classification Codes: C22, C58, D81

1. Introduction

The stock market has been growing at a fast pace around the world since the 18th century. The famous New York Stock Exchange(NYSE) was established at that time, which made North America the largest stock market in the world. What's more, the export of oil in Canada plays an essential role in the North American economy. It influences the demand and supply in internal areas and also impacts the global financial situation. Therefore, it is essential and practical to study the Canadian stock markets.

Secondly, the Canadian oil industry is a rather essential factor that influences the international economy. For example, the Canadian oil market has a high susceptibility to American tariff shocks. Meghan Potkins stated in his article "Canadian oilpatch angst ramps up amid falling oil prices, on-again, off-again tariffs" that the Canadian oil companies are hedging their prices and preparing for more volatility under the background of American Tariffs [1]. In the recent environment, institutions, organizations, and individuals should have learnt something from the past risks. A lot of people would complain about the bad economy nowadays and have no ideas to face the risk. Therefore, it quite worth a lot to studying the Canadian oil industry.

During the long period of stock market development, a large number of bad

events have occurred. For example, the Great Depression, the Asian Financial Crisis in 1997, the Subprime Mortgage Crisis in 2007, and the 2020 Stock Market Crash(Covid-19). However, related institutions and governments just started to operate strict regulations after the financial crisis. Furthermore, the global economy is rather hard and complex nowadays because of several kinds of virus attacks, war, and new American policies. It seems that people are living in a rather severe situation these days. Hence, it is rather important to research volatilities and risks in the historical stock market as well as predict risk events for the future. This research aims to compare different volatility models and predict the future volatility with the best fit one.

Current research has published many essential results. For example, Louis H. stated in his article 'How asymmetric is U.S. stock market volatility?' that the implied volatilities usually increase after a negative shock in the US stock market and the markets performs negative skewness [2]. Dinghai X. concluded that the Canadian stock market volatilities are more sensitive to good and bad news during the COVID-19 [3]. The volatility leverage and clustering effects are discussed in many publications, and they will be highly considered in this research.

In this paper, we are focusing on the Canadian oil stock market and using 3 main oil companies Suncor Energy, Imperial Oil, and Cenovus Energy since Canada is one of the most crucial oil-exported countries. The 3 companies covered almost all the up, middle, and downstream of the Canadian oil industry,

which means that the data from these 3 companies are representative and worth studying. Besides, the fluctuations in oil stock prices would influence the investors' attitudes towards Canadian dollars. For example, the oil demand will increase during the post-war period due to economic recovery. Then the oil price also increases which leads to a significant rise in related companies' stock price. Thus, studies on oil company stock prices could help to predict the economy around the world and enact regulatory measures for governments.

Volatilities predictions are one of the most important parts of financial risk management. There exists a lot of models which could capture the volatility in these years. Tim Bollerslev first established the generalized autoregressive conditional heteroskedasticity (GARCH) model in 1986 and it has a great ability to succinctly capture volatility clustering in financial rates of returns [4]. However, the GARCH model could not give an accurate estimation when explaining the structural changes in data generating process. This problem was raised by Diebold in 2003 [5]. What's more, it could not capture the different effects of past residuals on volatility in the data, which is the leverage effect. Therefore, the standard GARCH model could be inappropriate for this research's purpose. Then the threshold GARCH (TGARCH) created by Jean-Michel Zakoian in 1994 will be applied in this research [6].

However, the traditional TGARCH can only model the volatility of time series with leverage effect. It has a really simple model of mean, which neglects the autocorrelation of stock returns. What's more, a simple TGARCH(1,1,1) model

cannot predict the future volatility because the returns process are regarded as simple white noise. Therefore, the ARMA-TGARCH model will be applied in this research.

In terms of extreme risk analysis, the extreme value theory (EVT) is mainly used to study the tail distribution of financial asset returns [7]. This theory studies the extreme values picked and fits them into a Generalized Pareto Distribution (GPD). There are 2 main methods in EVT, Generalized Extreme Value (GEV) and Peaks Over Threshold (POT). The 2 methods picked extreme values in totally different ways and the reasons why POT is chosen will be discussed later in this paper.

Combined with the time series models and extreme value theory models, both the volatility and extreme risks from oil stock series was studied and they provides more complete and reasonable predictions for decision-makers.

2. Theory and models

2.1 Data source

The data of 3 companies' stock prices (Suncor Energy, Imperial Oil, and Cenovus Energy) are downloaded directly from Yahoo Finance and the closed prices are used for analysis. The data period starts at 2010-01-01 and ends at 2024-03-04, which includes an extreme event (The pandemic of COVID-19). The following studies how the return and volatility series moved over time.

2.2 Variables selection

The prices and returns variables have high correlations with their own lag terms and past residuals and therefore, the autoregressive and moving average (ARMA) and GARCH model are used in this research

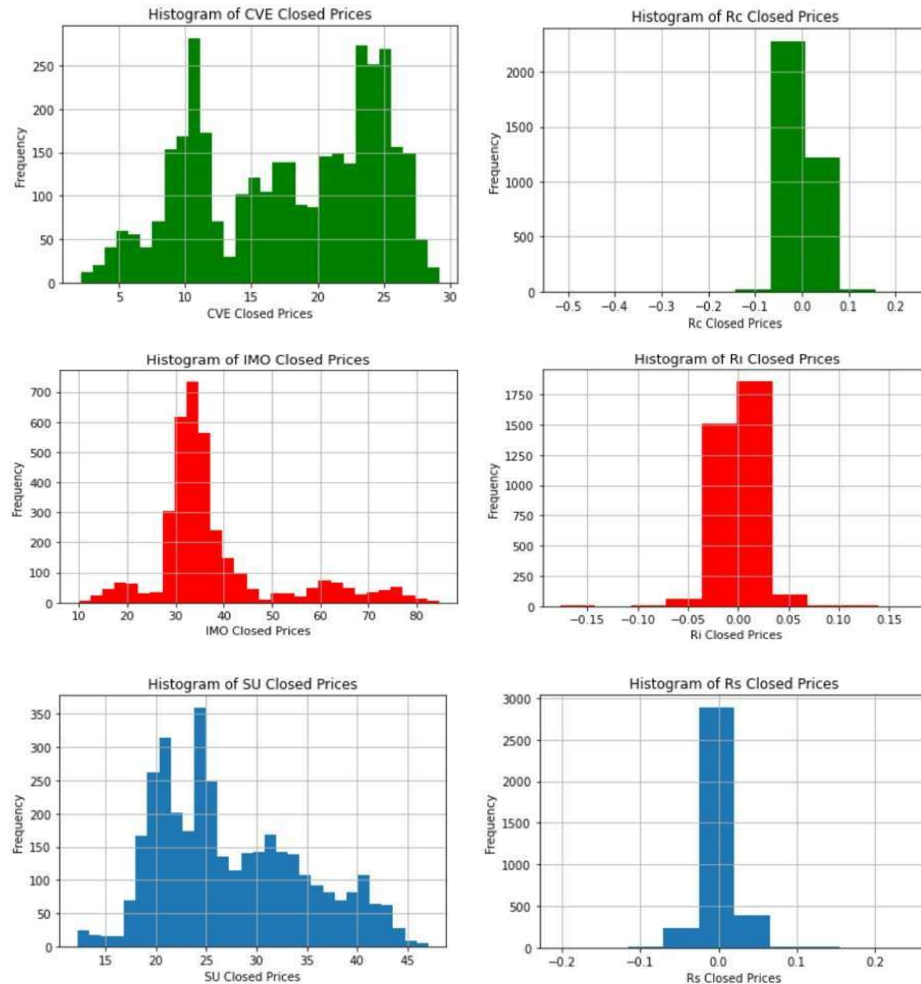


Fig. 1 Histograms of returns and stock prices in 3 companies

Figure 1 shows the distributions of 3 companies' stock prices and returns. It is obvious that the data variation is quite large in prices, while the returns are more concentrated. Then there exists the problem that the same change amounts in stock price might indicate different levels of fluctuations. Besides, the returns reflect the relative change which is more comparable. As a result,

the return series are chosen for better analysis.

Using the return series still has some problems, even if it is better than the price series. For instance, Figure 1, shows that almost all values are greater than -0.05 and less than +0.05, which is a rather small scale. This leads to some misestimations on volatility and returns. In this research, this problem was assumed to be ignored.

2.3 Data preparation

There is a second reason why the returns series should be chosen. One of the assumptions of the ARMA and GARCH model is that the series should be stationary. Then Augmented Dickey-Fuller (ADF) tests are applied to the stock prices of 3 companies and the results show that all of them are non-stationary. Then the continuously compounded returns are computed and an ADF test was performed on them. The Figure 2 plots below show the ADF statistics (in red line) and critical values (grey bars) at 10%, 5%, and 1% significance. They represent that the statistics are all smaller than the critical values, which means the return series are stationary.

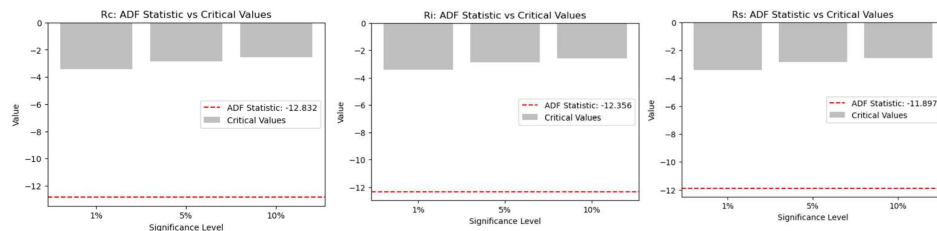


Fig. 2 ADF tests for 3 companies at different significant levels

	SU	IMO	CVE
mean	0.000206	0.000297	-0.000018
variance	0.000424	0.000348	0.000772
skewness	-0.141045	-0.624979	-4.720388
kurtosis	16.070254	15.656568	127.593509

Fig. 3 statistic summary for 3 companies.

Figure 3 shows the statistics summary for these 3 companies' returns. The mean values are all close to 0, which means that there is no systematic increase or decrease in stock prices. The small variance shows that the risks of oil stock are relatively small. This is inconsistent with the facts due to small scale of returns. Besides, the negative skewness shows that the stock has a higher downside risk, and the significant large kurtosis shows that the tail risks are extremely large. Overall, the returns series has the characteristic of heavy tail, negative skewness, and the return data are extremely concentrated around 0.

2.4 Method introduction

In this paper, volatility is studied more than the return process, so a GARCH model is used for description and analysis. The standard GARCH(1,1) model is expressed as:

$$y_t = \mu + s_{t-1}, \text{ where } s_t \sim N(0, 1)$$

$$\sigma_t^2 = \mu + \alpha s_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (1)$$

In order to better fit the data with leverage and clustering effects, a threshold generalized autoregressive conditional heteroskedasticity (TGARCH) model will be applied here. In this model, a threshold term is added in the volatility equation, which increase the effect of negative past residuals (s_{t-1}) and do not

change the effect of positive past residuals. The TGARCH(1,1,1) model is expressed as:

$$\begin{aligned}
 y_t &= \mu + s_{t-1}, \text{ where } s_t \sim N(0, 1) \\
 \sigma_t^2 &= \mu + \alpha s_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma s_{t-1}^2 \cdot I(s_{t-1} < 0), \text{ where} \\
 I &= \begin{cases} 1, & s_{t-1} < 0 \\ 0, & \text{otherwise} \end{cases}
 \end{aligned} \tag{2}$$

The standard TGARCH model did not give specific description of the mean returns, and it cannot provide detailed predictions for the future since the residuals today cannot be obtained for future. Then an ARMA-TGARCH model can easily solve the problems. A standard ARMA(1,1)-TGARCH(1,1,1) model can be expressed as:

$$\begin{aligned}
 y_t &= \phi y_{t-1} + s_t + \theta s_{t-1}, \text{ where } s_t \sim N(0, 1) \\
 \sigma_t^2 &= \mu + \alpha s_{t-1}^2 + \beta \sigma_{t-1}^2 + \gamma s_{t-1}^2 \cdot I(s_{t-1} < 0), \text{ where} \\
 I &= \begin{cases} 1, & s_{t-1} < 0 \\ 0, & \text{otherwise} \end{cases}
 \end{aligned} \tag{3}$$

The formula (3) depicts both the mean and volatility series for the models and therefore, it can predict the future volatility by obtaining the future returns from mean model. This is the main model used in this research to analyze the risks in stock prices.

Secondly, an extreme value theory will be applied to the data, as there are a lot of extreme events during the data periods, including the pandemic of COVID-19. The main 2 methods of EVT are GEV (also called block maxima method) and POT (peaks over threshold). The GEV picks extreme values by blocking data into several parts and extracting the largest value in each part.

However, the POT method sets up a threshold first and then picks all values that are larger than that threshold. In this research, the POT method is applied because the data were extremely clustered, especially around 2020 (Pandemic of Covid-19). Then GEV is not accurate here. The traditional EVT is used on return series and the excess returns will follow GPD (Generalized Pareto Distribution), where GPD is a distribution that describes excess values. However, in this research, the focus is on volatility, so I am applying POT to the volatility series directly. I chose the threshold which is 97% of the conditional volatility. Later, the excess volatility follows a generalized Pareto distribution and outputs 3 parameters: shape, location, and scale. Then some further analysis will be conducted based on this.

3. Results and discussion

3.1 Standard TGARCH model

The standard TGARCH model was first applied to 3 companies' data in order to capture the volatility in financial markets and show some leverage effect of the data. Then the output summaries from TGARCH model are listed below:

Mean Model					
	coef	std err	t	P> t	95.0% Conf. Int.
mu	-5.3042e-06	2.791e-04	-1.900e-02	0.985	[-5.524e-04,5.418e-04]
Volatility Model					
	coef	std err	t	P> t	95.0% Conf. Int.
omega	8.7564e-06	5.356e-12	1.635e+06	0.000	[8.756e-06,8.756e-06]
alpha[1]	0.0100	6.246e-03	1.601	0.109	[-2.242e-03,2.224e-02]
gamma[1]	0.1000	1.969e-02	5.078	3.814e-07	[6.140e-02, 0.139]
beta[1]	0.9200	3.035e-03	303.154	0.000	[0.914, 0.926]

Fig. 4 TGARCH model for Suncor Energy

Mean Model					
	coef	std err	t	P> t	95.0% Conf. Int.
mu	9.6201e-04	2.654e-06	362.479	0.000	[9.568e-04,9.672e-04]
Volatility Model					
	coef	std err	t	P> t	95.0% Conf. Int.
omega	7.0647e-06	1.535e-12	4.601e+06	0.000	[7.065e-06,7.065e-06]
alpha[1]	0.0500	1.418e-02	3.526	4.218e-04	[2.221e-02,7.779e-02]
gamma[1]	0.0500	2.674e-02	1.870	6.152e-02	[-2.413e-03, 0.102]
beta[1]	0.9050	9.561e-03	94.653	0.000	[0.886, 0.924]

Fig. 5 TGARCH model for Imperial Oil

Mean Model					
	coef	std err	t	P> t	95.0% Conf. Int.
mu	-8.4295e-05	3.478e-04	-0.242	0.809	[-7.660e-04,5.974e-04]

Volatility Model					
	coef	std err	t	P> t	95.0% Conf. Int.
omega	1.5904e-05	4.023e-12	3.953e+06	0.000	[1.590e-05,1.590e-05]
alpha[1]	0.0501	1.671e-02	3.000	2.702e-03	[1.737e-02,8.286e-02]
gamma[1]	0.1190	4.874e-02	2.442	1.462e-02	[2.348e-02, 0.215]
beta[1]	0.8716	2.357e-02	36.988	1.759e-299	[0.825, 0.918]

Fig. 6 TGARCH model for Cenovus Energy

From the output tables, in Suncor's model, the PVALUE of gamma and beta are smaller than 0.05, which means that they are quite significant. The volatility depends on past volatility but may not depend on past residuals. The results are rather similar in these 3 models. However, in Imperial Oil's model, the TGARCH model fits the data really well. All PVALUE are significantly small (<0.05), which means there truly is some leverage effect in the data. Then the Imperial Oil data will be used for further analysis.

3.2 ARMA-TGARCH model

Initially, a proper model with the best orders should be selected. As it is known to all that ARMA(1,1) and GARCH(1,1) are usually the best models, the similar models ARMA(0,0) – TGARCH(1,1,1), ARMA(1,1) – TGARCH(1,1,1) and ARMA(1,1)-TGARCH(1,1,2) are applied on the returns process.

	TGARCH(1,1)	ARMA(1,1) – TGARCH(1,1,1)	ARMA(1,1)- TGARCH(1,1,2)
AIC	-18163.9	-19497.4	-18169.9
BIC	-18151.6	-19466.5	-18145.2

Table 1 AIC and BIC of different ARMA-TGARCH models

Table 1 gathers all AIC and BIC values of different models, and after minimizing the information criteria, there comes the conclusion that ARMA(1,1)-TGARCH(1,1,1) is the best model.

In order to better fit the Imperial Oil return data, an ARMA-TGARCH model is applied here. The mean model is an ARMA(1,1) process and the volatility model is a TGARCH(1,1,1). This captures the effect of past returns, volatility, and residuals on today's return and volatility. The picture below shows the output of ARMA(1,1) – TGARCH(1,1,1) is expressed below.

Mean Model					
	coef	std err	t	P> t	95.0% Conf. Int.
mu	-3.4117e-04	2.486e-04	-1.372	0.170	[-8.285e-04,1.462e-04]

Volatility Model					
	coef	std err	t	P> t	95.0% Conf. Int.
omega	7.0450e-06	4.327e-10	1.628e+04	0.000	[7.044e-06,7.046e-06]
alpha[1]	0.0500	6.789e-03	7.365	1.768e-13	[3.669e-02,6.331e-02]
gamma[1]	0.0500	2.741e-02	1.824	6.815e-02	[-3.727e-03, 0.104]
beta[1]	0.9050	5.384e-03	168.092	0.000	[0.894, 0.916]

Fig. 7 ARMA-TGARCH model for Imperial Oil

The output shows that PVALUES of all alpha, beta, and gamma are extremely small, and the sum of ARCH and GARCH parameters is close to 0. This represents that past residual and volatility have significant effects on the volatility. What's more, the volatility process has high persistence. It means that the risks have a strong and persistent effect on the oil stock market. Additionally, it also shows that there will be large volatility in the future if there is a great

shock today. This is consistent with the clustering effect in the return process.

In the mean model, it proves that the returns of the oil stock price have a high correlation with its own lag terms. The short-term returns can be forecasted since it is not white noise. The model almost fully captures the leverage effects and volatility clustering. This result is consistent with the conclusion that good news always come with good news and bad news come with bad ones.

In terms of model selection, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) are used in this model. From the model output, we can see that the AIC is -19497.4 and the BIC is -19466.4. Both of the information criteria are significantly small, which means that the model fits the data well.

In order to find the real volatility, one should use all the stock prices in one day to compute the variance on that day. However, there are several hundred thousand pieces of data in this research, and it will raise computational problems. Therefore, it is assumed that the standard deviation of the past 20 days at time t is the realized volatility. Then we can see whether the model estimates volatility correctly.

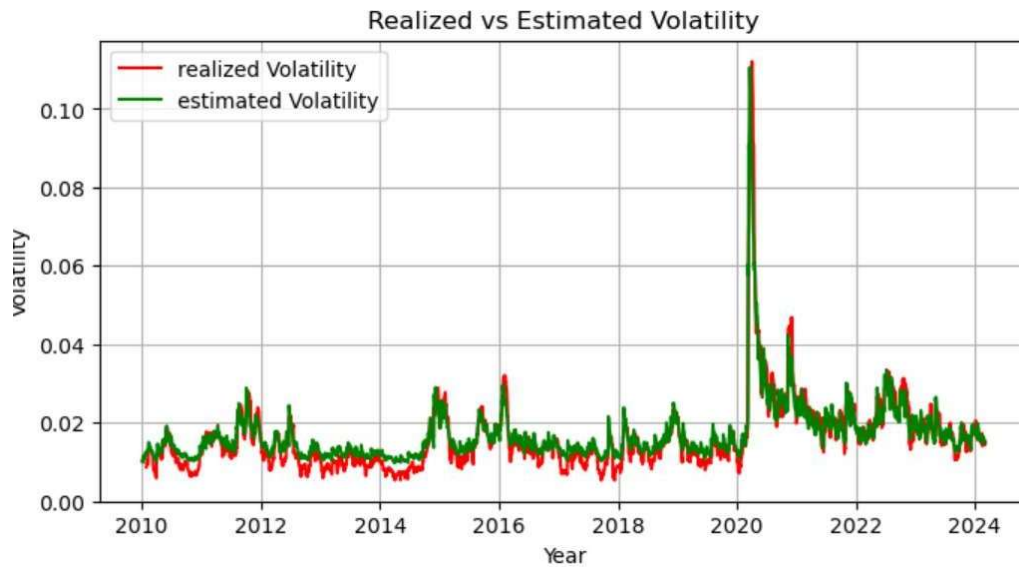


Fig. 8 Comparison between realized and estimated volatility

From the plot, we can see that the volatility series was fluctuating in a small interval for most of the time. This shows that oil stocks do not have much risks in regular times. We can also see that the green line (estimated volatility) fits the data really well except for some of the extreme and negative values. Besides, during the pandemic of COVID-19, the volatility experienced a significant increase. The size was almost 5 times larger than the regular volatility. In reality, the oil stock returns fluctuated for a long while. At that time, a lot of companies went bankrupt due to lack of regulations. This also proves that the negative residuals could have high effects on volatility.

Now that the model was really good, the predictions are necessary. Instead of computing 262 days' (from 2024-03-05 to 2025-03-05, there are 262 business days) predictions directly, this research used rolling windows of size 3555 to train an ARMA-TGARCH model and predict the next day's value of volatility for 262 times.

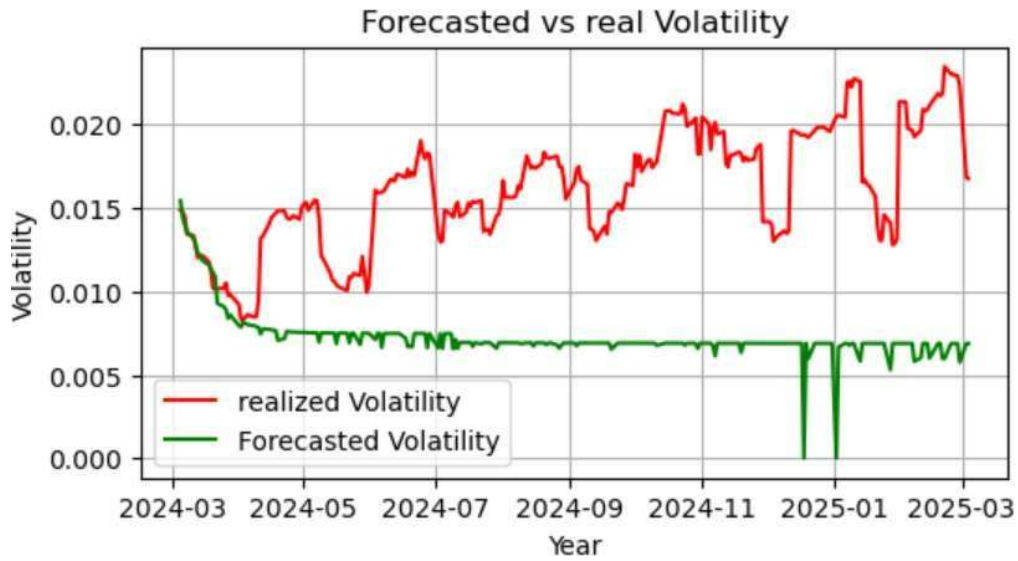


Fig. 9 Forecasted vs. real Volatility

Figure 9 represents the comparison between the volatility estimated from the ARMA-TGARCH model and real volatility. From the plot, we can see that the predicted volatility follows a great decrease at the first time and then remains stable. The short-term predictions are rather good, but long-term ones are not. The predictions are extremely flat after that and there exists some sudden decrease or increase in volatility. The main reason is that rolling windows forecasting leads to the prediction far from normal values. There are also some problems with the rolling windows forecasting. The window size was set at 3555, which is too large to be sensitive enough to new data. This might cause the volatility to stay flat for a long time. Besides, this method fits an ARMA-TGARCH model 262 times (prediction period), which is extremely computational. The estimated trends are also the opposite sometimes. In a word, the prediction from the ARMA-TGARCH model is not good enough and some further analysis should be done for better predictions.

3.3 EVT-POT

With non-satisfactory ARMA-TGARCH prediction, EVT-POT is used to study tail events. EVT-POT stands for extreme value theory and peaks over threshold. It studies the extreme value behavior from historical data and fit them into some distributions. Then some implications are created from the distributions. In extreme value theory, the excess part of volatility will follow the generalized Pareto Distribution. However, the distribution does not fit the data well at 99% level. Then some parameter adjustments are performed. In this research, the threshold was set at about 90% of the conditional volatility, which is about 0.02 and from the plot above, it shows that 0.02 exceed almost all the regular volatility and remain some extreme values around 2020. Then the model outputs 3 parameters: shape, location and scale, which are 0.4, $1.1e-05$ and 0.004 respectively. The shape parameter describe the tail behaviors of the distribution and a positive shape parameter (0.4) which means that the data is slightly heavy tailed. The location parameter depicts the shift of the distribution on x-axis and a significantly close to 0 parameter means that the distribution starts where excess volatility equals to 0. The scale factor shows the spread of distribution and 0.004 means that the volatility above the threshold are clustered slightly.

The EVT-POT method used in this research is quite standard and basic. This makes the tail events analysis primary. The distribution only studies the tail behavior (in COVID-19) in historical data and apply it directly to the future. The

improvement advice will be mentioned in the conclusion.

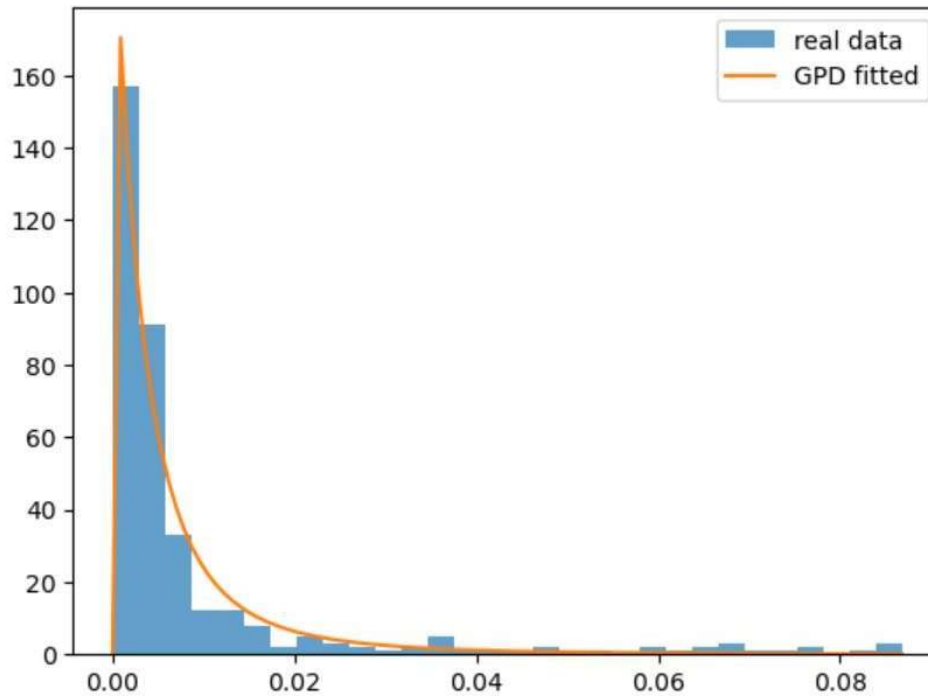


Fig. 10 GPD fitted excess volatility

From figure 10, it is obvious that the generalized pareto distribution fits the excess volatility well. This means that the tail behavior of original return data follows the assumption of GPD. And therefore, the fitted GPD can be used to describe the extreme volatility. After applying this distribution, a 99% value at risk based on historical data is about 0.04. This is a rather large volatility value since almost 99% of the past data are below the value. Furthermore, the future potential volatility would be rather large.

4. Conclusion

The data used in this research is the closed price of oil companies' stock, and it lasted from 2010-01-01 to 2024-03-04. The stock price varies a lot during

the past 15 years, and it has characteristic trends and seasons. Then returns series are analyzed in GARCH and EVT model.

Summarizing from the model, it is obvious to get the conclusion that ARMA-TGARCH model could predict short-term volatility, but highly underestimated long-term data. It could also be easy to get the point that this model could fit the data well, but in some extreme cases, ARMA-TGARCH is not proper. The basic model could only capture the leverage and clustering effect in data series, which should be improved. Moreover, the conclusion from extreme value theory indicates there is 99% possibility that the future volatility will not exceed 0.04.

This study also highlights that there should be some advice based on this research. For example, the high VaR could be some basic level of investment. For most investors, one could hedge the high-level volatility with some hedging assets such as the put options as well as hold some defensive assets such as gold and America Bonds. For foreign exchange traders, they could enter a long trade in Canadian dollars and for risk lovers, one could get into some financial products portfolios whose payoffs are high at both low and high stock price.

However, in terms of the primary method of extreme risk analysis, a risk prediction could not only depend on the past terms. Some economic factors such as climates, policies and stock trading volumes should be considered in the topic. These variables affects the stock's volatility in an exogenous way. For example, the climate change may influence the stock traders' incentives and then influence the trading strategies. Therefore, the ARIMAX(ARMA +

exogenous variables) and GARCH-X (GARCH + exogenous) model could be a better choice for risk management.

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